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Game Complete Analysis for Financial Markets Stabilization

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Aims and scope of the paper

The aim of this paper is to propose a methodology to stabilize the financial markets using Game Theory. Specifically, we will focus on two economic operators: a real economic subject and a financial institute (a bank, for example) with a big economic availability. For this purpose we will discuss about an interaction between the two above economic subjects: the Enterprise, our first player, and the Financial Institute, our second player.

Simbology

- S_0: *spot* price at time 0
- F_0: *forward* or *future price at time* 0
- *t*: expiration of the future or forward contract
- *i*: risk-free interest rate for the time t

Memorandum

- Hedging operation: purchase (or sale) of future contracts in order to reduce the exposure to specific risks on the market variables.

- Perfect hedge: complete elimination of the risks involved.

- The general relationship that links the future price and spot price is as follows:

$$F_0 = S_0(1+i)^t.$$

Presentation of the players

The Enterprise may choose a strategy $x \in [0,1]$, depending on if it intends

- not to hedge $(x = 0)$,
- to hedge partially $(0 < x < 1)$,
- to hedge totally $(x = 1)$.

Presentation of the players

The Financial Institute may choose a strategy $y \in [-1,1]$, depending on if it intends

- to purchase on the market of the underlying ($y > 0$),
- to sell on the market of the underlying ($y < 0$),
- not to intervene on the market of the underlying ($y = 0$).

TERMINOLOGY

- M_1: quantity of goods that the Enterprise has to buy at time 1
- M_2: economic availability of Frances expressed in goods
- S_0: spot price of the underlying at time 0.
- S_1 (y): spot price of the underlying at time 1. It's given by

$$S_1(y) = (S_0 + ny)\ (1+i)$$

TERMINOLOGY

- x: percentage of goods purchased by the Enterprise through futures
- y: percentage of goods that the Financial Institute buys or sells on the spot market of the underlying asset
- F_0: future price at time 0.
- $F_1(x,y)$: future price at time 1. It is given by

$$F_1(x,y) = S_1(1+i)^t + mx.$$

- $(1+i)^{-t}$: discount factor at the rate i for the time t.

Payoff function of the Enterprise

The payoff function of the Enterprise:

$$f_1(x,y) = M_1 (1- x) (F_0 - S_1(y))$$

Simplifying we have:

$$f_1(x,y) = -v_1 y M_1 (1-x). \qquad v_1 = n(1+i)$$

- *Profit on goods without hedge*

Payoff function of the Financial Institute

The payoff function of the Financial Institute:

$$f_2(x,y) = yM_2[F_1(x,y)\,(1+i)^{-t} - (S_0 + ny)\,(1+i)],$$

Simplifying we have:

$$f_2(x,y) = yM_2\mu_1 x \qquad \mu_1 = m(1+i)^{-t}$$

- *Profit on the future market*
- *Tax of the normative authority for markets stabilization*

Critical space of the game

Determinant of the jacobian matrix:

$$det\ J_f(x,y) = M_1 M_2 v_1 y \mu_1 x + M_1 M_2 \mu_1 (1-x)\ v_1 y$$

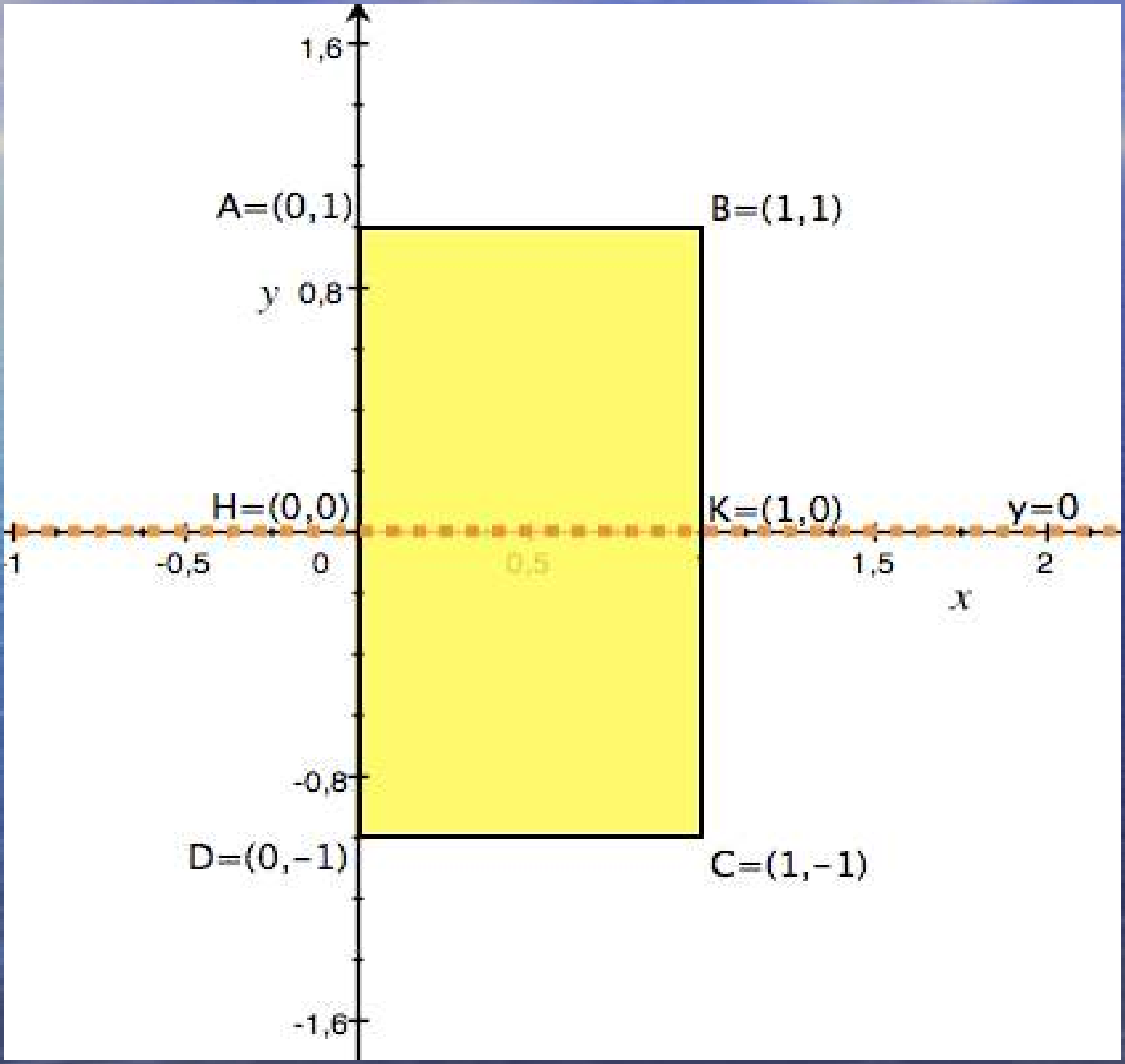
Critical space of the game:

$$Z_f = \{(x,y) : M_1 M_2 v_1 y \mu_1 x + M_1 M_2 \mu_1 (1-x)\ v_1 y = 0\}$$

Doing the calculations:

$$Z_f = \{(x,y) : y = 0\}.$$

Critical space of the game



Biwin space

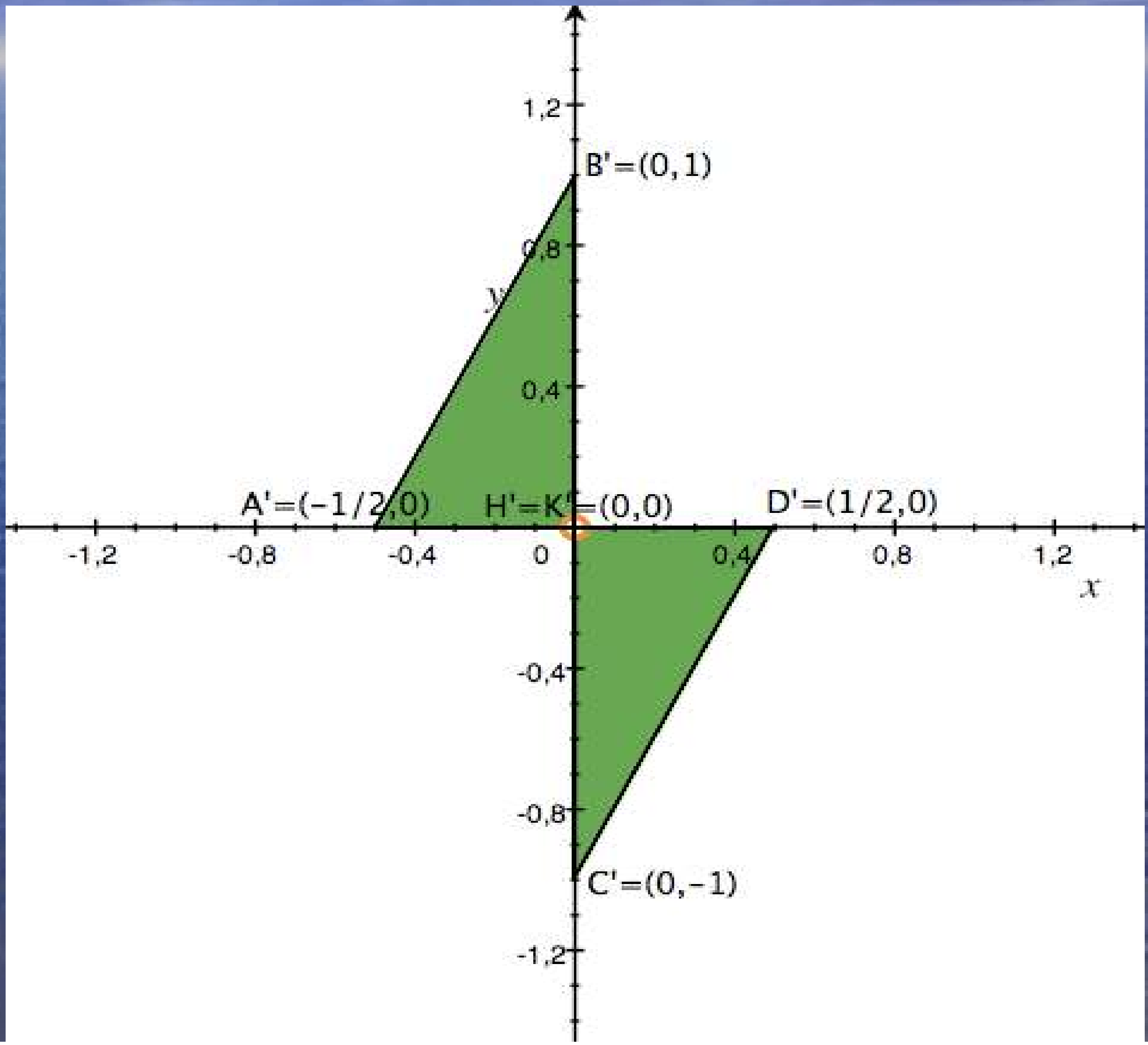
We suppose $M_1 = 1$, $M_2 = 2$ and $\nu_1 = \mu_1 = 1/2$.

f (A) = A'= (-1/2,0)

f (B) = B'= (0,1)

f (C) = C'= (0,-1)

f (D) = D'= (1/2,0)



Friendly phase

$\alpha = (1/2,1) \notin f(E\times F)$.

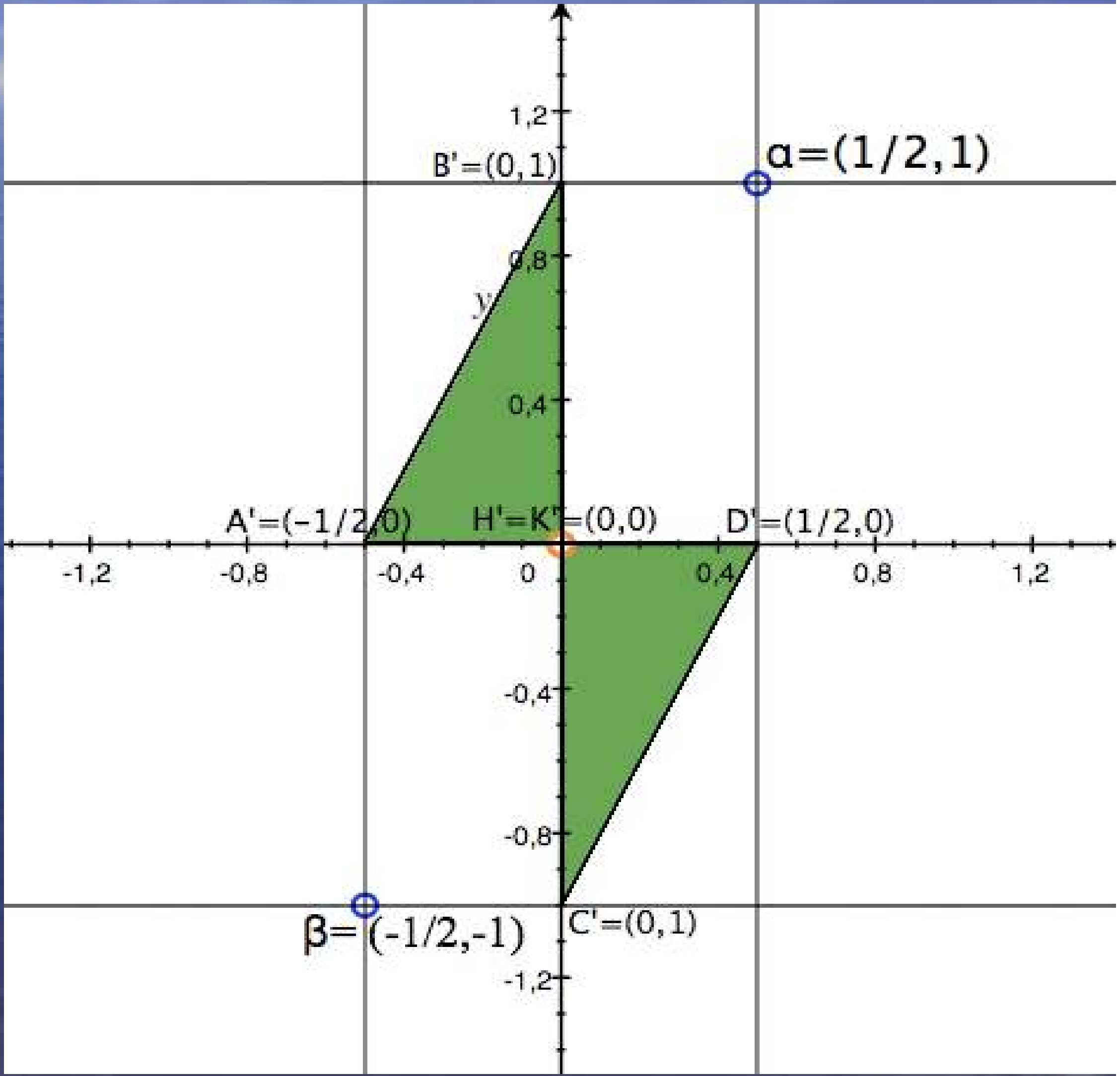
$\beta = (-1/2,-1) \notin f(E\times F)$.

$\partial^* f\,(E\times F) = \{B', D'\}$

$\partial^*_f\,(E\times F) = \{B\,,\,D\}$.

$\partial_* f\,(E\times F) = \{A', C'\}$

$\partial_{*f}\,(E\times F) = \{A\,,\,C\}$.



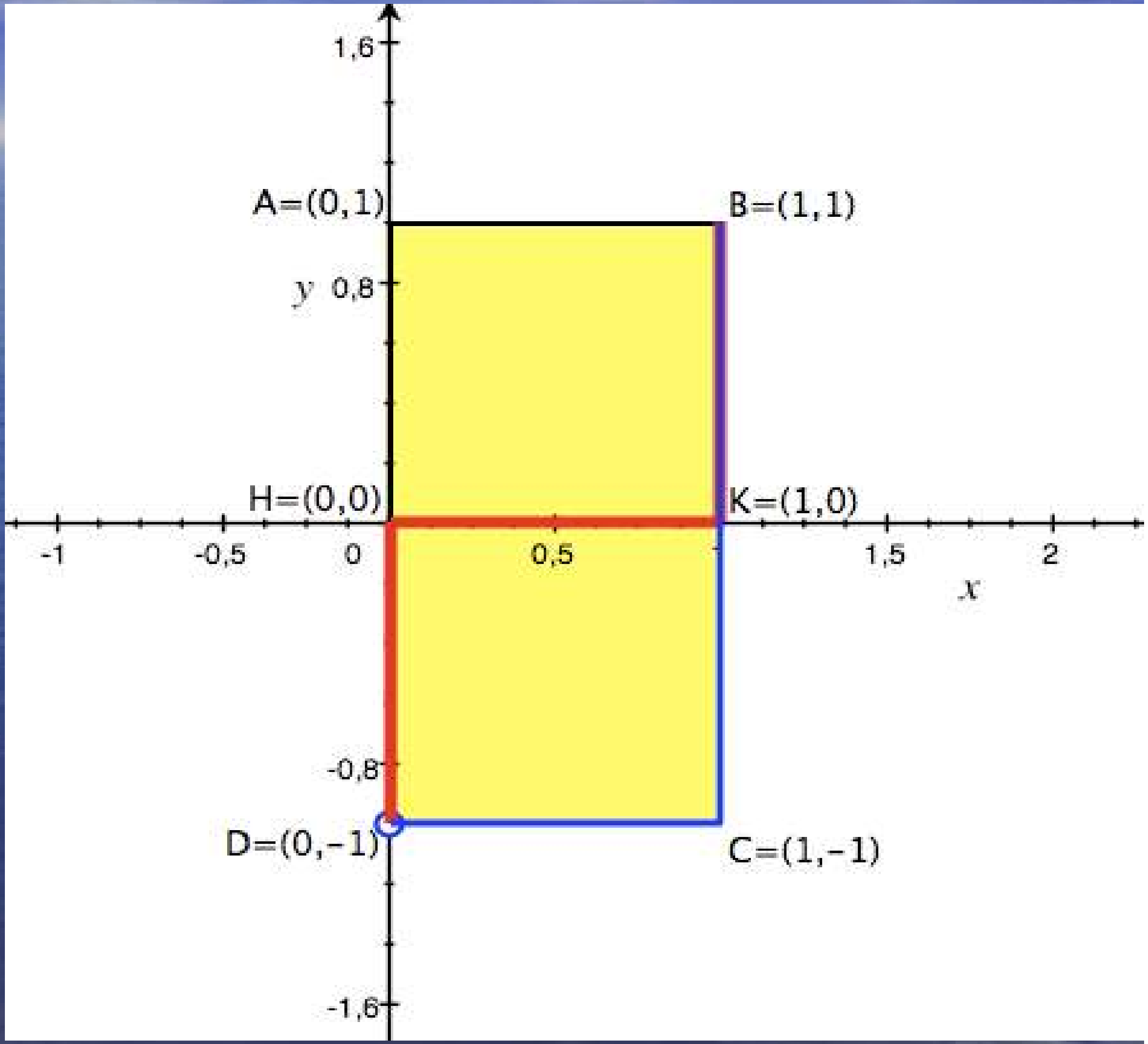
Equilibria of devotion

$f_1(x,y) = M_1[-\nu_1 y(1-x)]$

$f_2(x,y) = M_2\mu_1 xy$

$L_2(x) = \begin{cases} -1 \text{ se } 0 \leq x > 1 \\ F \text{ se } x = 1 \end{cases}$

$L_1(y) = \begin{cases} 1 \text{ se } y > 0 \\ E \text{ se } y = 0 \\ 0 \text{ se } y < 0 \end{cases}$



Eq $(L_1,L_2) = \{(0,-1)\} \cup [BK]$.

Nash equilibria

$f_1(x,y) = M_1[-\nu_1 y(1-x)]$

$f_2(x,y) = M_2\mu_1 xy$

$\partial_1 f_1 = M_1\nu_1 y$

$M_1\nu_1 y > 0$

$B_1(y) = 1$ se $y > 0$

$B_1(y) = E$ se $y = 0$

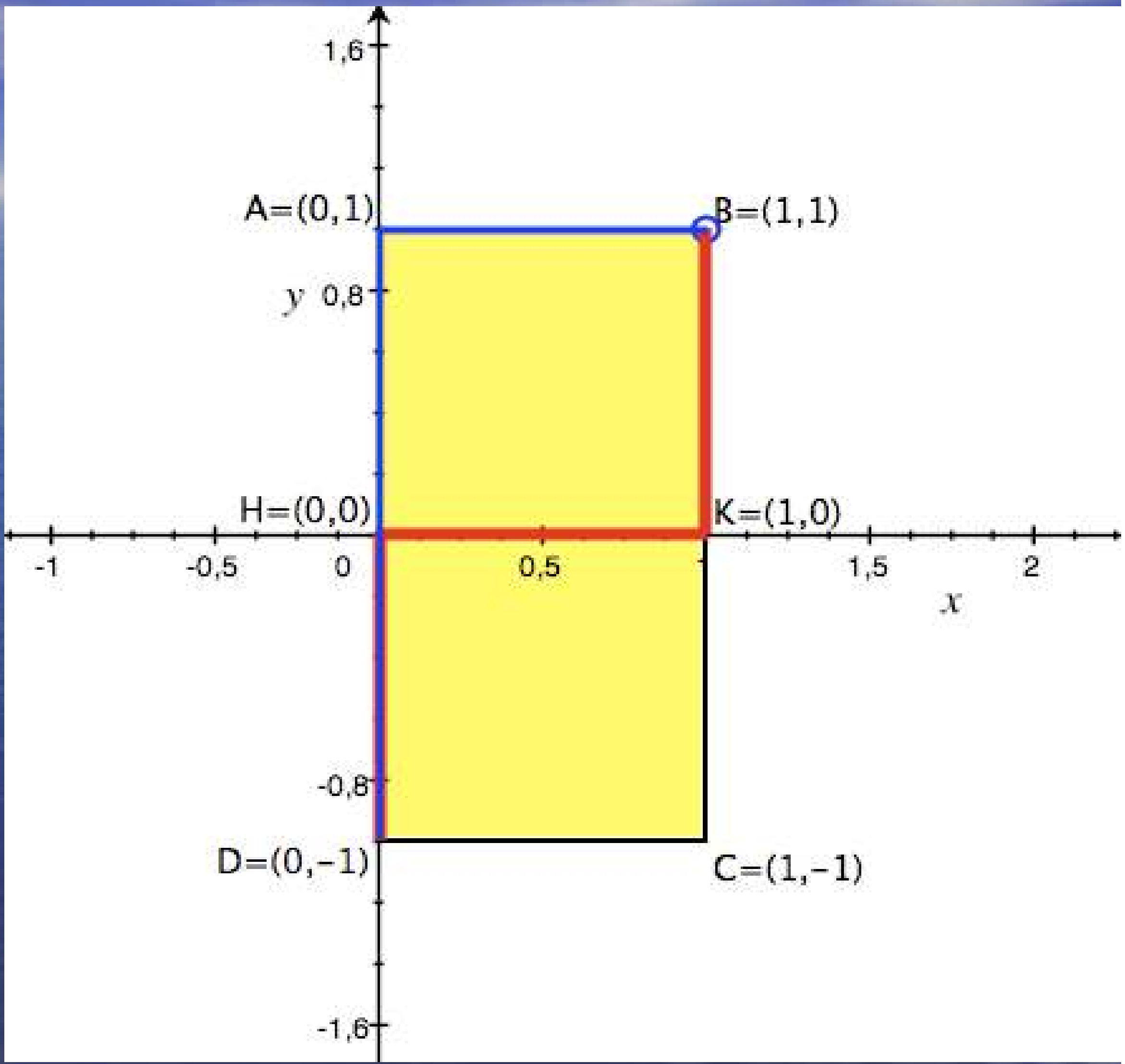
$B_1(y) = 0$ se $y < 0$

$\partial_2 f_2 = M_2\mu_1 x$

$M_2\mu_1 x > 0$

$B_2(x) = 1$ se $x > 0$

$B_2(x) = F$ se $x = 0$



$Eq(B_1,B_2) = \{(1,1)\} \cup [HD]$.

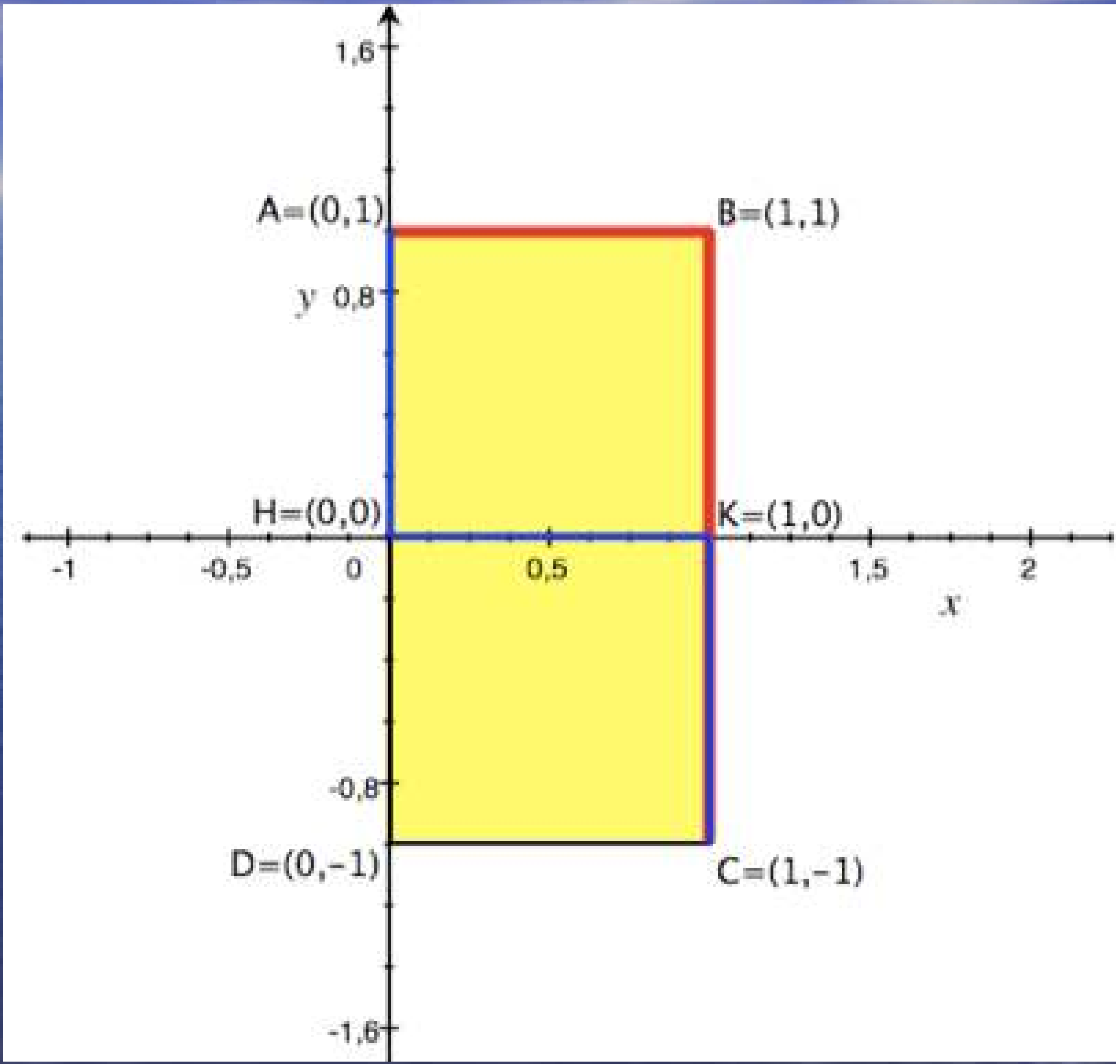
Offensive equilibria

$f_1(x,y) = M_1[-\nu_1 y(1-x)]$

$f_2(x,y) = M_2\mu_1 xy$

$$O_2(x) = \begin{cases} 1 \text{ se } 0 \leq x < 1 \\ F \text{ se } x = 1. \end{cases}$$

$$O_1(y) = \begin{cases} 0 \text{ se } y > 0 \\ E \text{ se } y = 0 \\ 1 \text{ se } y < 0. \end{cases}$$



$Eq(O_1,O_2) = \{(0,1)\} \cup [KC].$

Defensive bistrategies

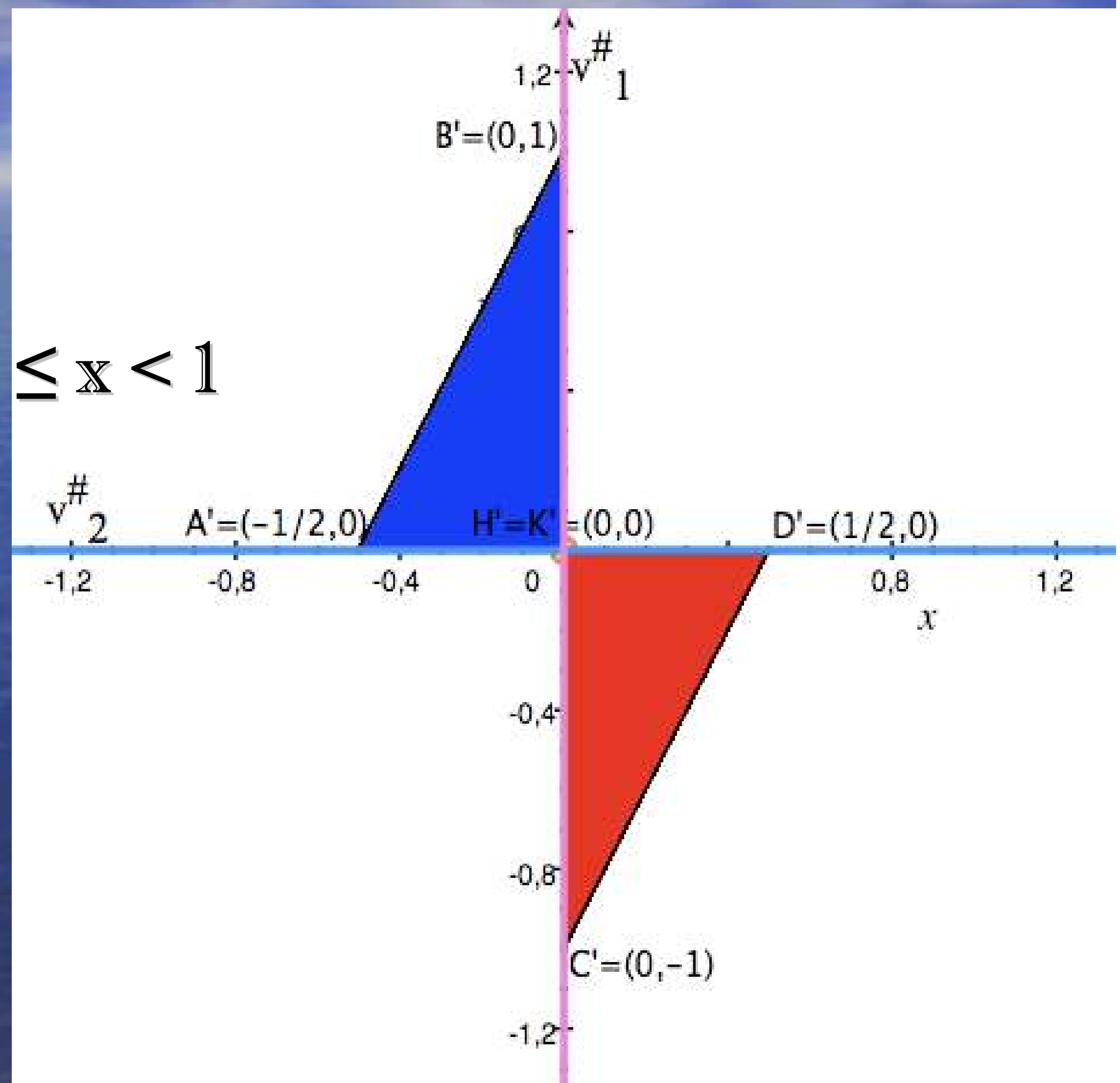
$f_1(x,y) = M_1[-\nu_1 y(1-x)]$
$f_2(x,y) = M_2\mu_1 xy$

$v^{\#}_1 = \sup_{(x\in E)} f_1^{\#} = 0$

$f_1^{\#} =$ $M_1[-\nu_1(1-x)]$ se $0 \leq x < 1$

0 se $x = 1$

$x^{\#} = 1$



$v^{\#}_2 = \sup_{y\in F} f_2^{\#} = 0$

$f_2^{\#} =$ 0 se $y \geq 0$

$M_2\mu_1 y$ se $y < 0$

$y^{\#} \geq 0$

$(x^{\#}, y^{\#}) = [BK]$

Cooperative solutions

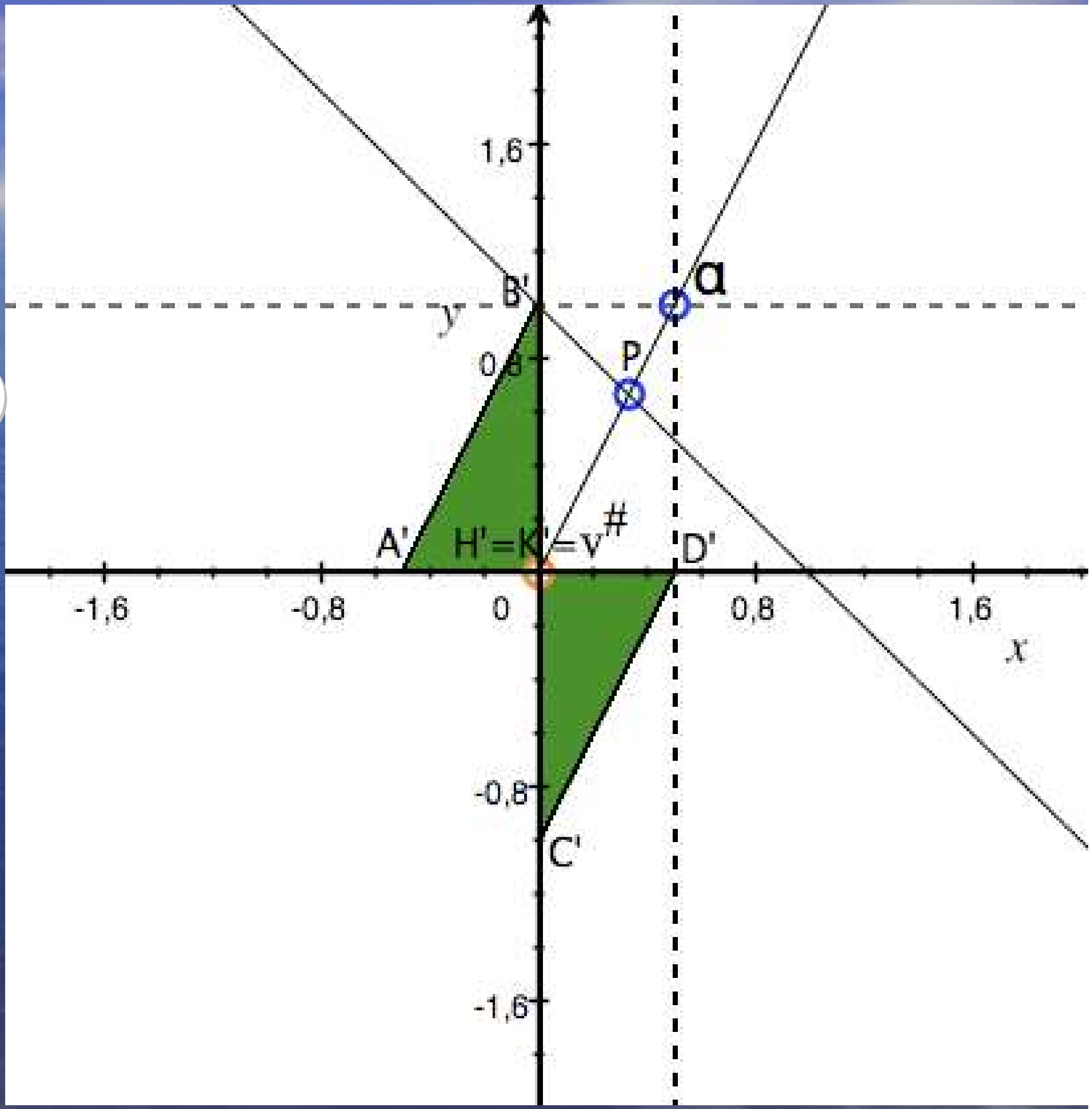
$f_1(x,y) = M_1[-\nu_1 y(1-x)]$
$f_2(x,y) = M_2\mu_1 xy$

sup $\partial^* f(E\times F) = (1/2,1)$

inf $\partial^* f(E\times F) = (0,0)$.

$X+Y = 1$
$Y = 2X$.

$P = (1/3,2/3)$



Conclusions

The game just studied suggests a possible regulatory model that provides the stabilization of the financial market through the introduction of a tax on financial transactions. In fact, in this way it is possible to avoid speculation by which our modern economy is constantly affected. The only optimal solution is the cooperative one, in which both players win something. In fact, all non-cooperative solutions lead to mediocre results for at least one of the parties.

Innovative reading key

Now it is possible to provide an interesting key in order to understand the conclusions which we reached using the transferable utility solution. Since the point B = (1,1) is also the most likely Nash equilibrium, the value 1/3 (that the Financial Institute pays by contract to the Enterprise) can be seen as the fair price paid to the Enterprise by the Financial Institute to be sure that they arrive effectively to more likely Nash equilibrium B = (1,1), which is also the optimal solution for the Financial Institute.

THANKS FOR ATTENTION